Delay-Phase Precoding to Alleviate Beam Defocus Effect for Circular Arrays

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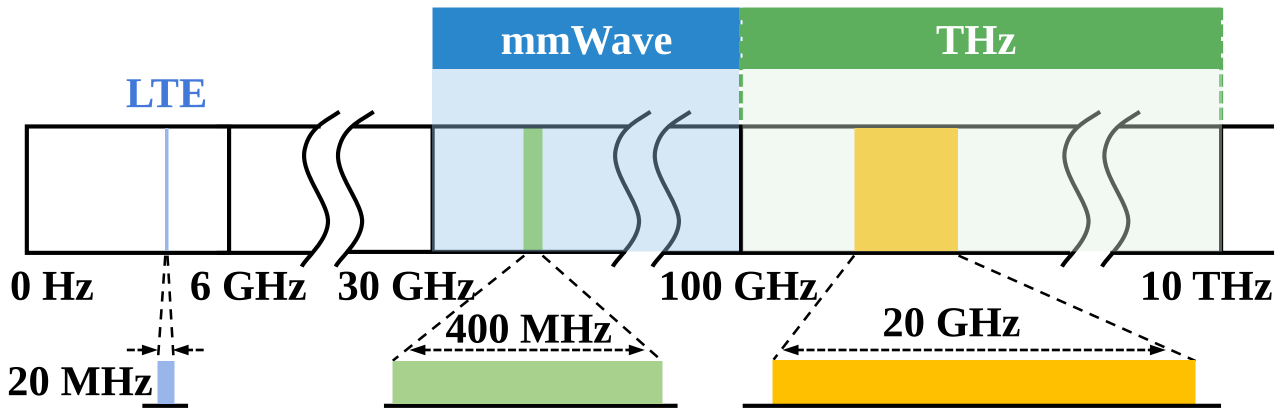
Z. Wu and L. Dai, "Delay-phase precoding to alleviate beam defocus effect for circular arrays," in *Proc. 2023 IEEE Global Commun. Conf. (IEEE GLOBECOM'23),* Kuala Lumpur, Malaysia, Dec. 2023.

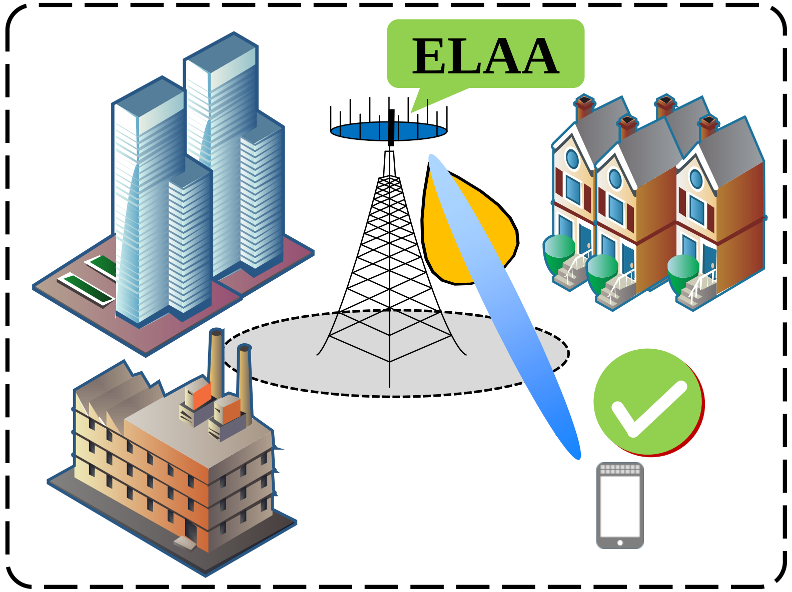
Outline

- Background
- From Beam Split to Beam Defocus
- Delay-Phase Precoding Algorithm
- Summary

Terahertz Communication with ELAA

- To satisfy the target peak data rate, terahertz (THz) band with abundant frequency resources has been viewed as one of the enabling technology for 6G communications
- Advantage: In THz band (0.1-10 THz), up to 20 GHz bandwidth could realize Tbps data rates
- Challenge: To compensate for the high attenuation of THz (160GHz: ~80dB/km), extremely large-scale antenna array (ELAA) becomes essential to form enough beamforming gain

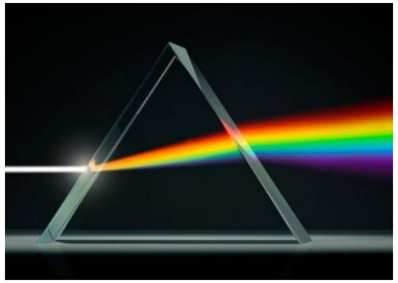




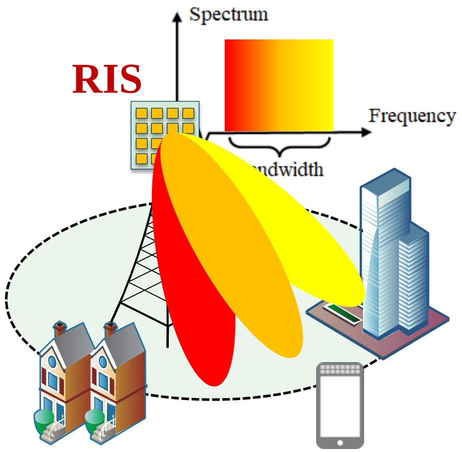
THz Wideband + ELAA: Beam Split Effect

- **Beam split effect with uniform linear array (ULA)**
  - **Beams split into different directions at different frequencies**
  - **Beams at non-central frequency could not be correctly aligned, leading to deteriorative beamforming gain at desired direction**

Prism



Dispersion of light



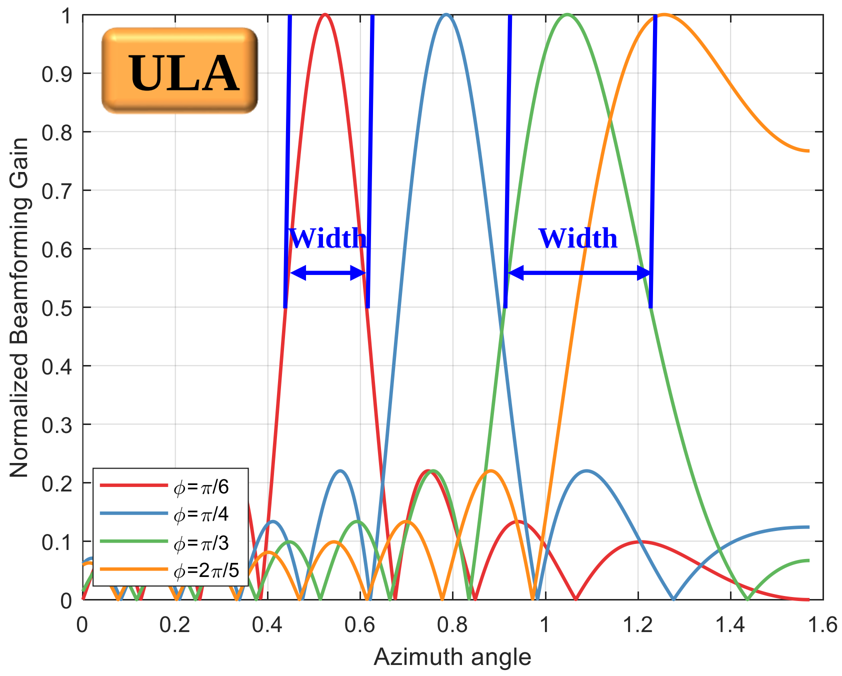
Beam Split Effect

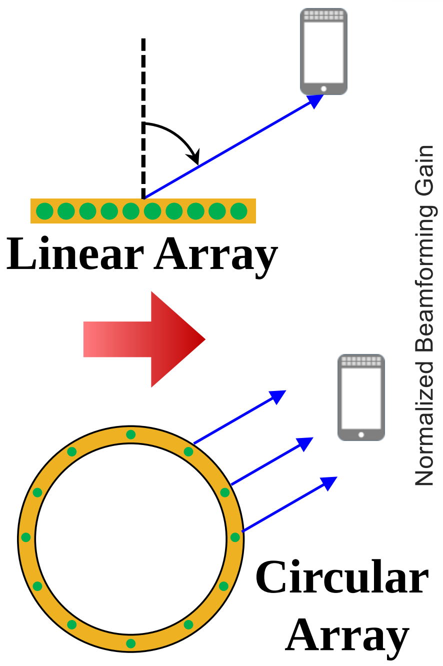
Parameters	Beam Width	Beam Shift	Relative Shift
Frequency: 30 GHz Bandwidth: 2 GHz Array: 16×16	11.25°	3°	26%
Frequency: 30 GHz Bandwidth: 2 GHz Array: 60×60	3°	3°	100%
Frequency: 100 GHz Bandwidth: 20 GHz Array: 16×16	11.25°	9°	80%
Frequency: 100 GHz Bandwidth: 20 GHz Array: 60×60	3°	9°	300%

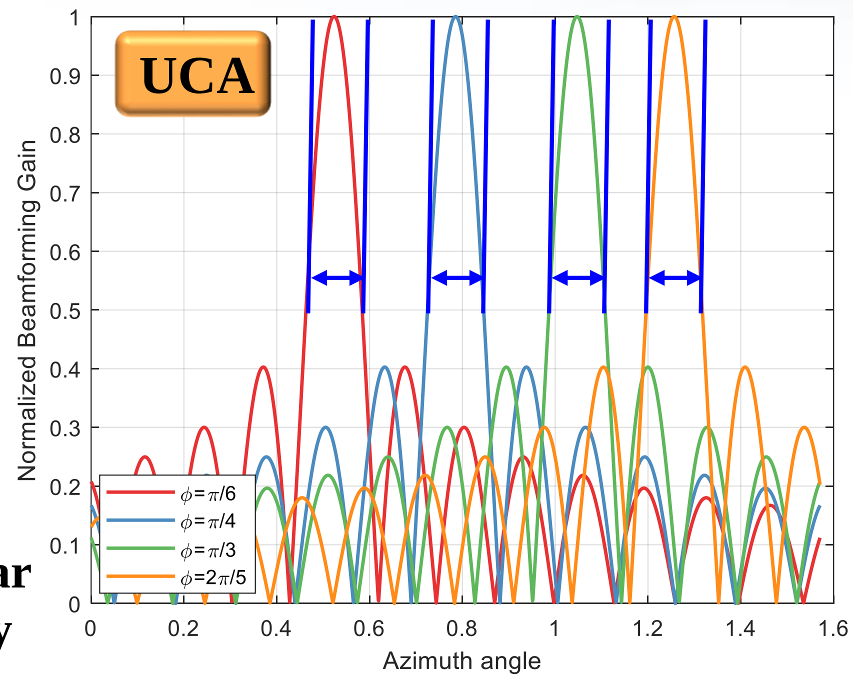
L. Dai, J. Tan, Z. Chen, and H. Vincent Poor, "Delay-phase precoding for wideband THz massive MIMO," *IEEE Trans. Wireless Commun.*, vol. 21, no. 9, pp. 7271-7286, Sep. 2022.

From Linear Array to Circular Array

- **Comparison between ULA and uniform circular array (UCA)**
  - **ULA: The beam pattern is distorted near the end-fire of ULA**
  - **UCA: With uniform beam pattern, UCA could provide better coverage, improved ergodic capacity and better bit error rate (BER)[1, 2]**







[1] W. Tan and S. Ma, "Antenna array topologies for mmWave massive MIMO systems: Spectral efficiency analysis," *IEEE Trans. Veh. Technol.*, vol. 71, no. 12, pp. 12 901–12 915, Dec. 2022.
[2] X. Cheng, Y. He, and J. Qiao, "Channel modeling for UCA and URA massive MIMO systems," in *Proc. Int. Conf. Comput., Netw. And Commun. (ICNC)*, 2020, pp. 963–968.

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Mechanism of Beam Split Effect with ULA

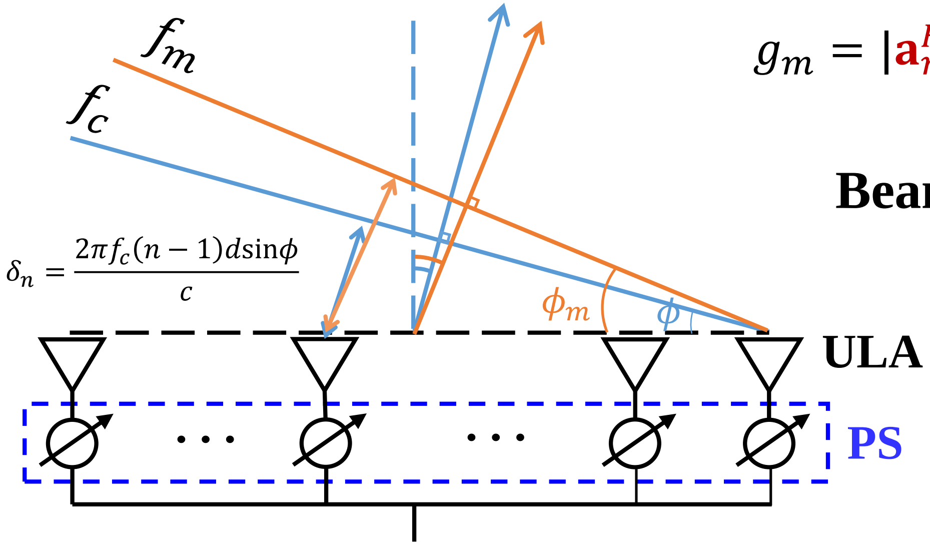
- **Wideband beamforming with PS-based hybrid precoding**
  - **Beamforming: Constructive interference in desired directions**
  - **Steering vector for N-element ULA at f_m is**

Frequency-dependent

$$\mathbf{a}_m(\phi) = \frac{1}{\sqrt{N}}\left[1, \exp\left\{j\frac{2\pi f_m}{c}d\sin\phi\right\}, \cdots, \exp\left\{j\frac{2\pi f_m}{c}(N-1)d\sin\phi\right\}\right]^T$$

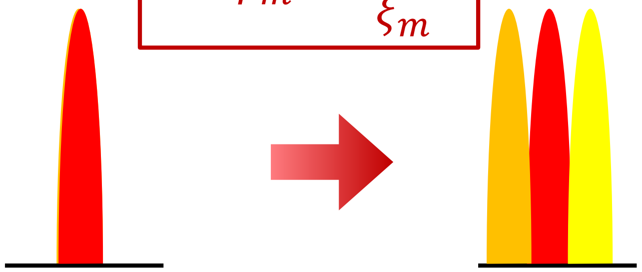
  - **In hybrid precoding, phase shifter (PS) could only form frequency-independent phase shifts**
  - **PS is designed at central frequency f_c with beamforming vector $\mathbf{a}_c(\phi)$**

Ideal beamforming



$$g_m = |\mathbf{a}_m^H(\phi_m)\mathbf{a}_c(\phi)| = 1 \iff f_m \sin\phi_m = f_c \sin\phi, \quad n = 1, \cdots, N$$

Beamforming direction: $\sin\phi_m = \frac{\sin\phi}{\xi_m}$ $\quad \xi_m = \frac{f_m}{f_c}$



Designed Beams **Split Beams**

From Beam Split to Beam Defocus

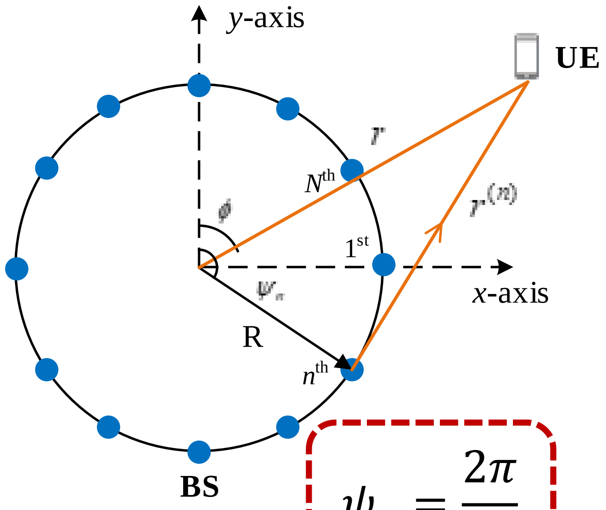
- **Wideband beamforming in UCA system**

Frequency-dependent

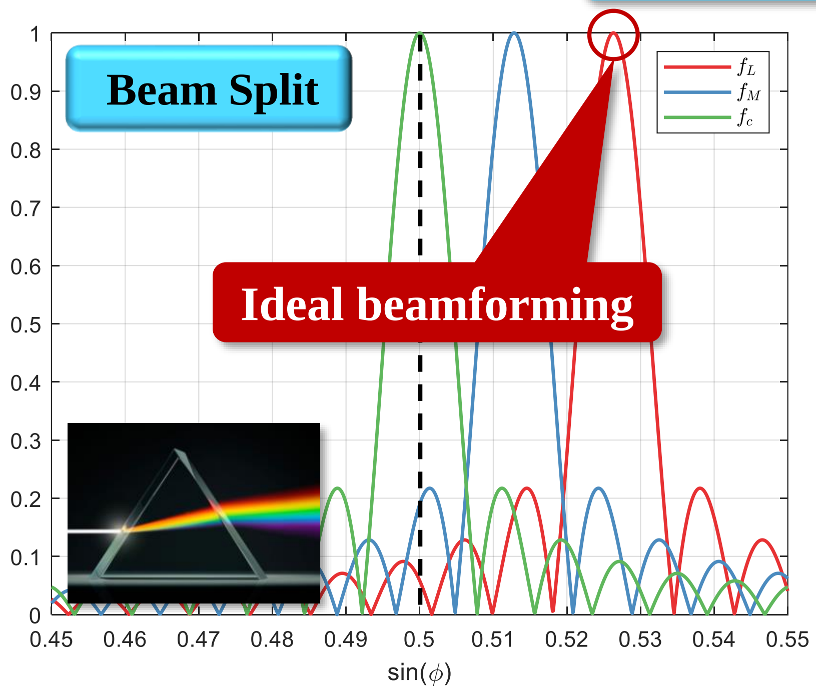
Steering vector: $\mathbf{a}_m(\phi) = \frac{1}{\sqrt{N}}\left[\exp\left\{j\frac{2\pi R}{c}f_m\cos(\phi-\psi_0)\right\},\cdots,\exp\left\{j\frac{2\pi R}{c}f_m\cos(\phi-\psi_N)\right\}\right]^T$

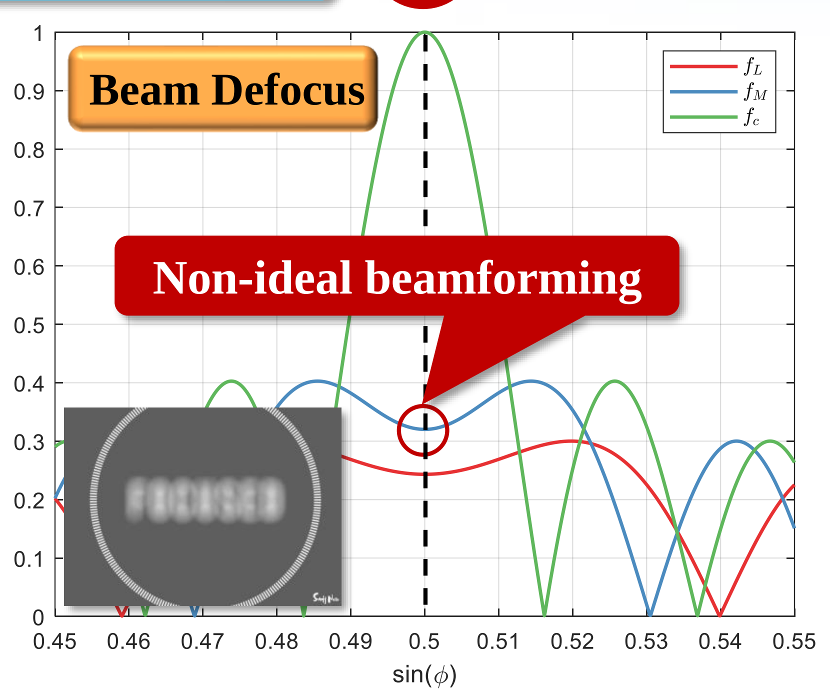
$$g_m = |\mathbf{a}_m^H(\phi_m)\mathbf{a}_c(\phi)| = 1 \Longleftrightarrow f_c\cos(\phi-\psi_n) = f_m\cos(\phi_m-\psi_n),\quad n=1,\cdots,N$$

No ϕ_m satisfies the equation for each n



$$\psi_n = \frac{2\pi}{n}$$





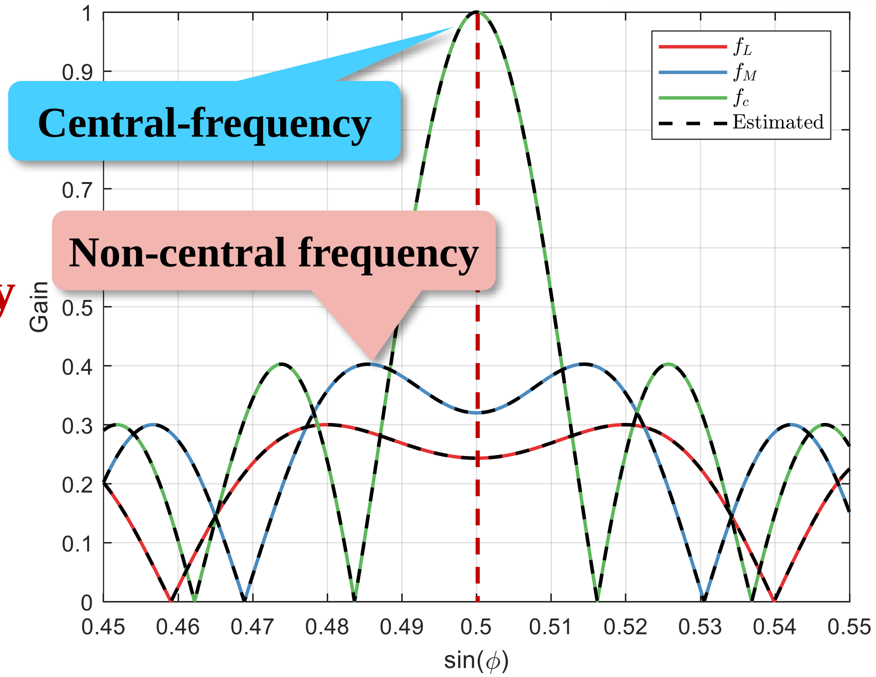
Beampattern in Angular Domain

Lemma 1: If frequency-independent beamforming vector $a_c(\phi_0)$ is employed, the achieved beamforming gain at frequency f_m at any direction ϕ could be expressed as

$$g = |\mathbf{a}_m^H(\phi)\mathbf{a}_c(\phi_0)| \approx \left| J_0\left(\frac{2\pi R}{c}\sqrt{f_m^2 + f_c^2 - 2 f_m f_c \cos(\phi - \phi_0)}\right)\right|$$

where $J_0(\cdot)$ denotes the zero-order Bessel function of the first kind.

- **Observations**
  - **Theoretical analysis matches well with calculations**
  - **Condition for ideal beamforming: $f_m = f_c$ and $\phi = \phi_0$**
  - **If $f_c \neq f_m$, ideal beamforming could not form in any direction in the same plane of UCA**



High-gain beams disappear in the same plane of UCA due to the beam defocus effect

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Delay-Phase Precoding (DPP)

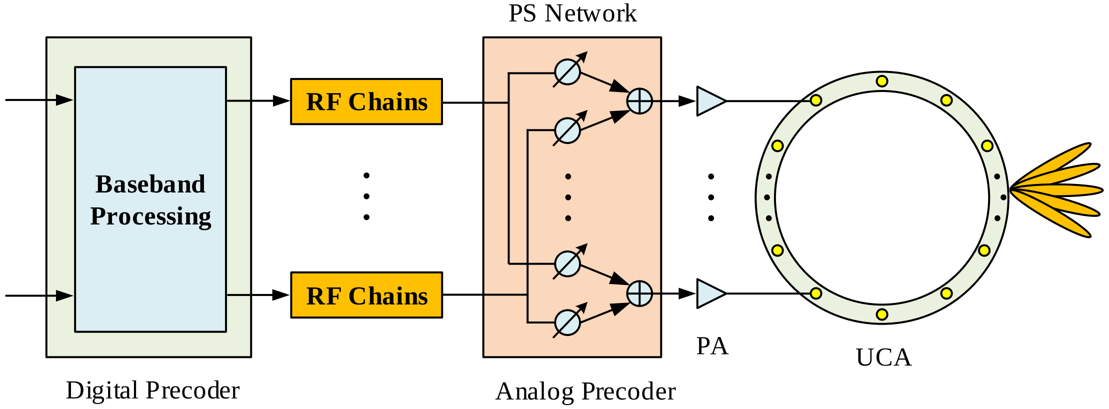
- **Beam split and beam defocus share the same reason: mismatch of generated frequency-independent phase shifts and required frequency-dependent phase shifts**
- **True-time delay (TDD) could form frequency-dependent phase shifts**

Frequency-Independent

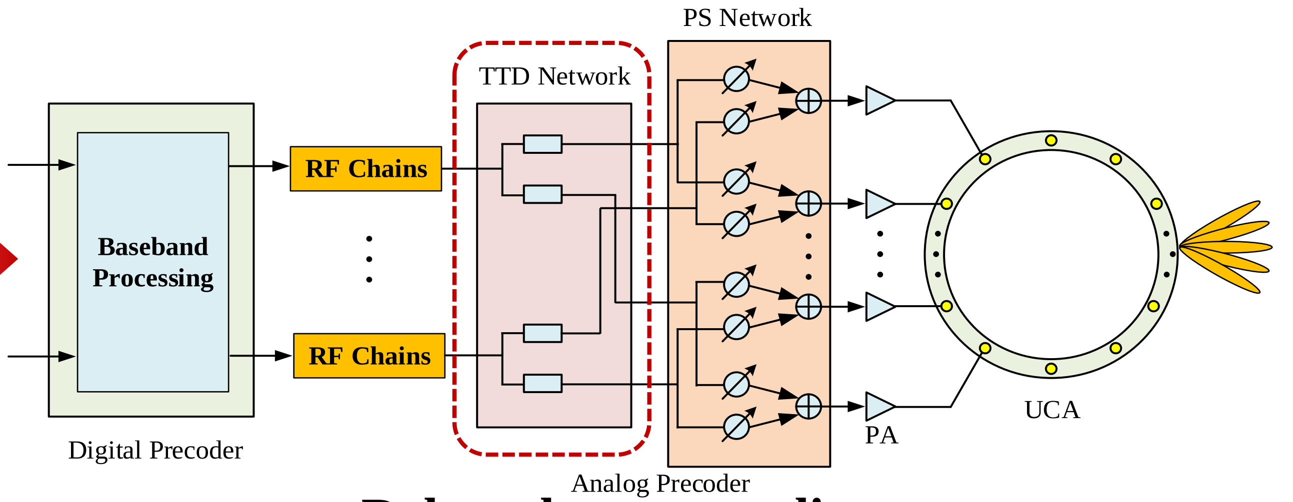
Phase shifts of PS: $\delta_n^{\mathrm{PS}} = \frac{2\pi R}{c} f_c \cos(\phi - \psi_n)$

Frequency-Dependent

Phase shifts of TTD: $\delta_n^{\mathrm{TTD}} = 2\pi t f_m = \frac{2\pi R}{c} f_m \cos(\phi - \psi_n)$



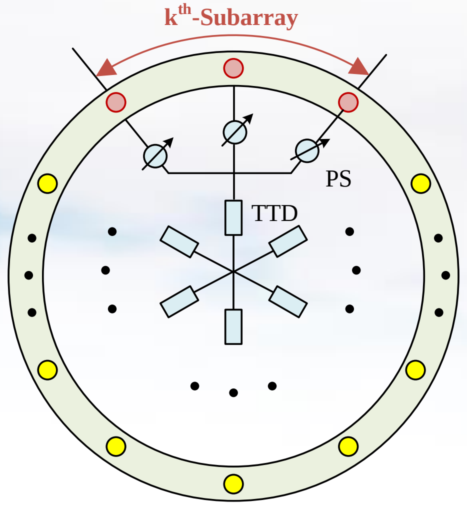
PS-based hybrid precoding



Delay-phase precoding

DPP to Mitigate Beam Defocus Effect

- **System model with the DPP architecture**



PS-based analog precoder

TTD-based analog precoder

$$\mathbf{y}_m = \mathbf{H}_m^H \mathbf{F}_{\mathrm{A}}^{\mathrm{PS}} \mathbf{F}_{\mathbf{A},m}^{\mathrm{TTD}} \mathbf{F}_{\mathrm{D},m} \mathbf{s}_m + \mathbf{n}_m$$

Lemma 2: If each RF chain connects to K TTDs and each TTD connects to a subarray with $P = N/K$ antennas. The optimal $\mathbf{f}_{l,m}^{\mathrm{TTD}}$ to maximize the beamforming gain could be expressed as

Center of subarray

$$\left(\mathbf{f}_{l,m}^{\mathrm{TTD}}\right)_k = \exp\left\{j\frac{2\pi R}{c}(f_m - f_c)\cos(\phi_l - \bar{\theta}_k)\right\} \quad \textbf{where} \quad \bar{\theta}_k = \frac{2\pi k}{K} + \frac{(P-1)\pi}{N}$$

The beamforming gain with DPP architecture could be written as $G_m(\mathbf{f}_l, \phi_l) \approx \frac{1}{P}\sum_{p=0}^{P-1} J_0(R_p)$

where $R_p = \frac{2\sqrt{2}\pi R}{c}(f_m - f_c)\sqrt{1 - \cos\left(\frac{(2p+1)\pi}{N} - \frac{\pi}{K}\right)}$ $\quad p = 0,1,\cdots,P-1$

Achieved gain

DPP Algorithm

- **Hybrid precoding design in UCA system**

Algorithm 1 DPP algorithm for UCA.

Input: Channel $\mathbf{H}_m$ and angles ϕ_l

Output: Analog and digital precoders $\mathbf{F}_{\mathrm{A}}^{\mathrm{PS}}$, $\mathbf{F}_{\mathrm{A},m}^{\mathrm{TTD}}$, $\mathbf{F}_{\mathrm{D},m}$

```
Rearrange the order of channel components |g_1| ≥ |g_2| ≥ ··· ≥ |g_RF| and obtain corresponding {φ_1, ··· , φ_RF};
for l = 1, 2, ··· , N_RF do
    Construct the beam steering vector a_c(φ_l) by (3);
    for k = 1, 2, ··· , K do
        Determine phase shifts of PS f̃_{l,k} by (22);
        Determine delay for k^th TTD t̃_{l,k} by (20);
    end for
    Construct F̃_l = [f̃_{l,1}, ··· , f̃_{l,K}];
    Construct p̃_{l,m} = [e^{-j2π f_m t̃_{l,1}}, ··· , e^{-j2π f_m t̃_{l,K}}]^T;
end for
Construct F_A^PS = [blkdiag(F̃_1), ··· , blkdiag(F̃_{N_RF})];
Construct F_{A,m}^TTD = blkdiag([p̃_{1,m}, ··· , p̃_{N_RF,m}]);
Obtain the equivalent channel H_{eq,m}^H = H_m^H F_A^PS F_{A,m}^TTD with H_{eq,m}^H = U_{eq,m} Σ_{eq,m} V_{eq,m}^H;
Determine the digital precoder F_{D,m} = V_{eq,m} Λ;
return F_A^PS, F_{A,m}^TTD and F_{D,m}.
```

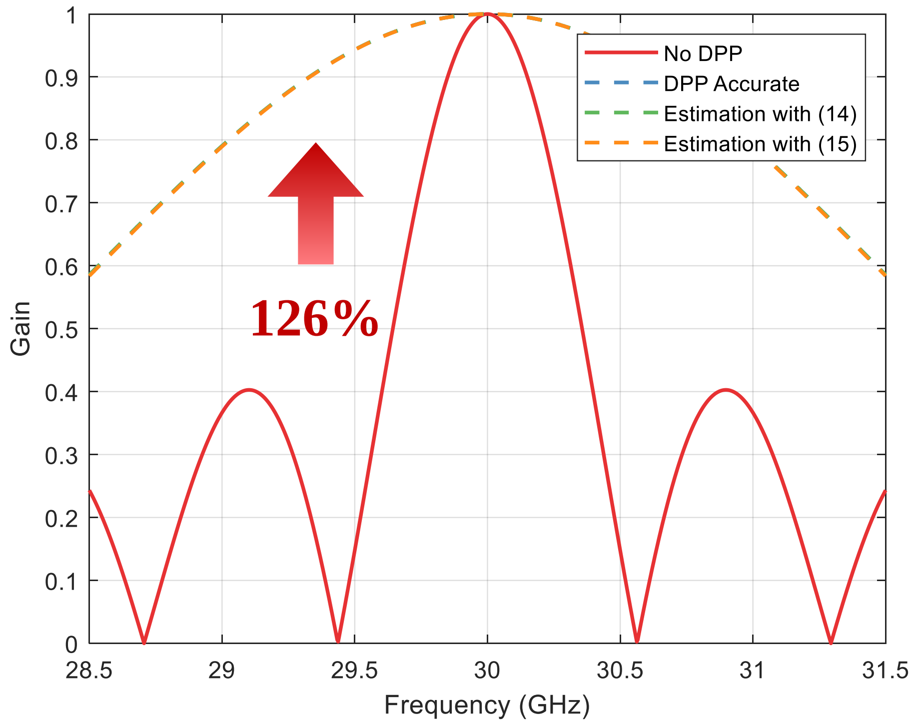
①**Sorting**: Rearrange the channel components in the descending order

②**Analog precoder**: Determine the phase shifts of PSs and delays of TTDs to construct the analog precoder

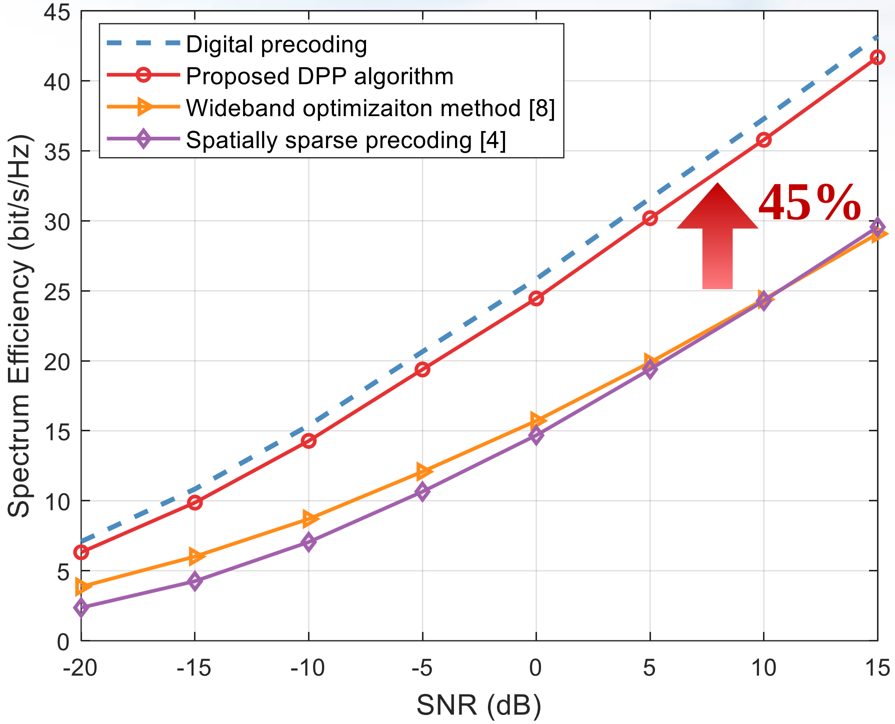
③**Digital precoder**: Determine digital precoder with SVD of the equivalent channel

Simulation Results

- **Parameters:**
  - **Number of half-wavelength antennas at TX $N_t = 256$, at RX $N_r = 4$, number of TTDs $K = 8$**
  - **Central frequency $f_c = 30$ GHz, bandwidth $B = 3$ GHz**



Comparison of beamforming gain



Comparison of spectrum efficiency

[4] O. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, "Spatially-sparse precoding in millimeter wave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499–1513, Jan. 2014.
[8] S. Park, A. Alkhateeb, and R. W. Heath, "Dynamic subarrays for hybrid precoding in wideband mmwave MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 16, no. 5, pp. 2907–2920, May 2017.

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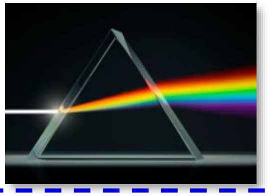
Conclusions

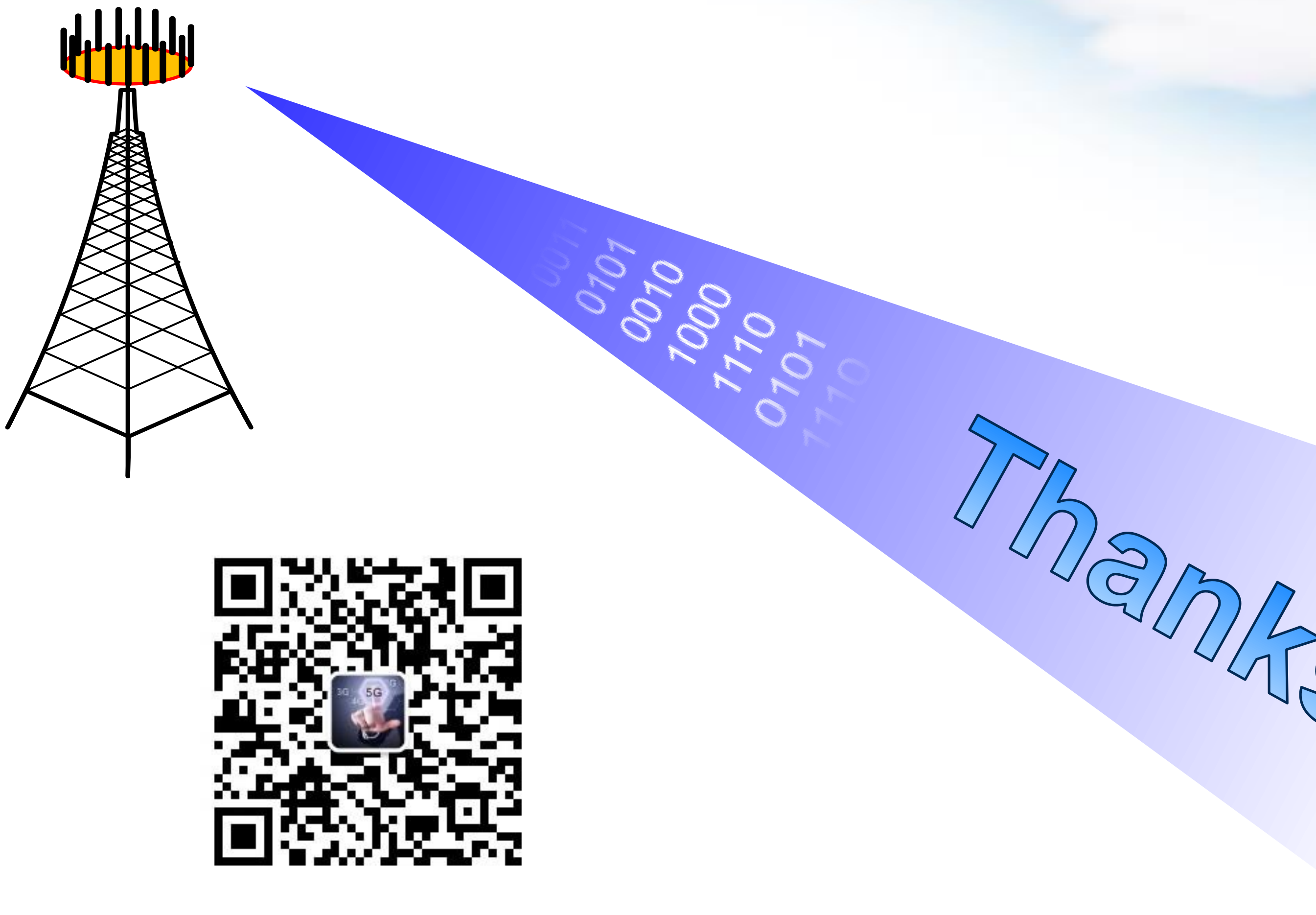
- **Beam Defocus Effect**
  - **Frequency-independent phase shifter (PS) could not generate required frequency-dependent phase shifts in wideband beamforming**
  - **Unlike beam split effect in ULA systems, the mismatch leads to beam defocus effect in UCA, where high-gain beams disappear at non-central frequency**



Beam Split Effect with ULA

Beam Defocus Effect with UCA

- **Delay-phase precoding (DPP) to alleviate beam defocus effect**
  - **True-time delay (TTD) is employed to generate frequency-dependent phase shifts**
  - **The principle to design TTD is to compensate at the center of subarray**
  - **Simulation results have verified the effectiveness of DPP architecture**



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